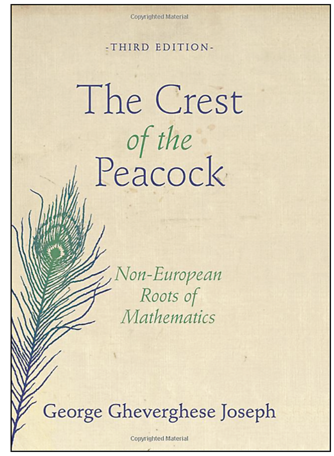
Book Review

The Crest of the Peacock: Non-European Roots of Mathematics

Reviewed by Clemency Montelle



The Crest of the Peacock: Non-European Roots of Mathematics
George Gheverghese Joseph
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For many *Notices* readers, *The Crest of the Peacock* needs no introduction. Now in its third edition, the book has enjoyed widespread popularity for providing an accessible account of the mathematical culture of non-European peoples. Responding to earlier critiques, Joseph's revised edition has some notable changes. For one, the subtitle has been modified. In a small but significant alteration, *Non-Western Roots of Mathematics* has been changed to *Non-European Roots of Mathematics*. Sections have been added and several removed, a reorganization that addresses various scholarly objections stemming from historical and authenticity concerns. Endnotes have been expanded so as to reflect some current research, and an enlarged bibliographic section that is now grouped primarily according to geographical region provides references for further reading. In the third edition it is stated that this book is intended to be "an effective resource for students and teachers of mathematics while remaining accessible to general readers." Indeed, Joseph's original intention was to appeal to the general public and teachers and students of mathematics, and in that aim he has enjoyed success. The book has been popular. It covers an assortment of interesting aspects of the mathematical activity of selected early cultures with a minimum of technical details in a humanistic, undemanding way. Joseph aims to summarize the essence of technical scholarship that would otherwise be out of reach of this audience.

The third edition opens with a lengthy preface in which Joseph discusses the underlying rationale for the book: a response to a field that he believes is overly dominated by the hegemony of a "Western version" of mathematics, or Eurocentrism. Such attitudes, Joseph points out, are epitomized by Morris Kline's statement that "the mathematics of Egyptians and Mesopotamians is the scrawling of children just learning to write, as opposed to great literature" (p. 175). While the majority of scholars would have no objection to Joseph's observation, it is well accepted nowadays that Kline's attitude, as well as others that Joseph invokes to support his argument,[1] is outdated. Historiography has altered a lot in the last quarter century. Scholarship in the history of mathematics has made an undeniable change of direction since the time in which these attitudes were prevalent. Since then research culture has moved increasingly away from a narrow monolithic portrayal of mathematics to a more inclusive picture that sees as valid and important a broad range of mathematical activity and is increasingly sensitive to wider contextual issues.

To be sure, *The Crest of the Peacock* deserves a share of credit for injecting momentum into these growing new attitudes, though in fact, these attitudes had been present for decades in the work of many devoted scholars. Otto Neugebauer brought serious historical treatment of non-western mathematics to the forefront beginning in the late 1930s and, along with numerous others, published

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[1] *Some of the opinions Joseph refers to were made over a century ago, such as Rouse Ball's book published in 1908.*

extensively throughout subsequent decades on various cultures of inquiry. Concurrently with the appearance of the first edition of *The Crest of the Peacock*, Ubiratan D'Ambrosio published a seminal work on ethnomathematics, a term he had coined in the late 1970s. This was followed closely by Marcia Ascher writing on the same topic. The mid-1990s saw the rise of books dedicated to bringing non-European mathematical traditions into mainstream scholarship. One example from the many instances will suffice: the work *Mathematics across Cultures* (2001), edited by Helaine Selin, presented historiographical reflections as well as expository articles on various mathematical traditions, including Mesopotamian, Egyptian, Islamic, Hebrew, Incan, Mesoamerican, Sioux Tipi and Cone, the Pacific Cultures, Aboriginal, Central and Southern African, West African, and Yoruba, Chinese, Indian, Japanese, and Korean! Indeed, nowadays a significant number of scholars are actively seeking to be less binary in their approach, and the most current research in these fields focuses on the interplay of ideas and trends and eschews a divisive attitude. Scholars are now reaping the benefits of a measured consideration of the many different cultures of inquiry in the history of mathematics; the associated themes of transmission, comparison, social context, situated cognition, and the flourishing of mathematical inquiry in cultures with contrasting epistemic priorities are proving fruitful, not only for history of mathematics but for the history and philosophy of science more broadly.

While Joseph passionately denounces the Eurocentric approach throughout his book, at times his authorial stance seems to perpetuate the divisions caused by these attitudes in the first place. For instance, his lack of a dedicated section on Greece (despite his frequent reference to Greek achievements and their transmission) is restrictive, particularly given the importance of this culture of inquiry for many others in the book, first and foremost the Islamic Near East. Indeed, his arguments in favor of historical inclusiveness would be better supported by discussing the relevance of the Greek mathematical culture as well. The polarity between "European" and "non-European" is not mitigated either by Joseph's dismissive references to (mostly unspecified) "western philosophers" (p. 35 and elsewhere), "western scholars", and "western historians of mathematics". One must be mindful of the fact that European scholars themselves were integral to or involved in critical advances in our knowledge of the so-called non-European cultures. "Western scholars" also deserve much credit for their historical work on diverse mathematical cultures. Joseph himself relies heavily on them in this book; an overwhelming number of entries in his bibliography are in English.

Joseph notes (p. xiii, for instance, and elsewhere) that "the concept of mathematics found outside the Greco-European praxis was very different." This is an undeniably important observation. One will note, however, that mathematical activity **in** those so-called Greco-European cultures was not monolithic either. The tendency for historians to overlook the various other strands of practice within these very cultures has led to regrettable biases. The emphasis on mathematics associated with elegant deductive style proposition-proof-type accounts over and above other mathematical activities has meant that these latter types of mathematics have been deemed less important or less relevant, and thus often left out or ignored in historical accounts. These activities include recreational, commercial, and utilitarian mathematics, and the wide spectrum of related practices such as teaching and the mathematics applied in other disciplines (such as the broader astral sciences, for instance). Various activities and byproducts in cultures that formed part of this Greco-European praxis (astrology, horoscopes, numerical tables, rough working, and so on) are just as important when it comes to giving a picture of the practice of mathematical activity in these cultures. Joseph is spot on when he comments that historians need to "confront historical bias, question the social and political values shaping the mathematics (and the writing of the history of mathematics), and search for different ways of 'knowing' or establishing mathematical truths in various traditions." But this sentiment is not to be limited to non-European mathematics. It is applicable to all cultures of mathematical inquiry.

Critical appraisal of scholarly sources can be complex, particularly when they are in conflict. One example Joseph confronts is the issue of the dating of the Bakhshālī manuscript, a work almost unique as a surviving physical exemplar of Indian mathematics from before the early modern period. Joseph notes (p. 358) that Kaye's assessment (made in 1933) that the manuscript belonged to the twelfth century CE is doubtful. He then instead appears to rely on an assessment of Hoernle[2] made in 1888, which estimates that the original was composed sometime in the early centuries of the common era. This is in light of Hayashi's recent analysis (1995), which locates the original text around the seventh century and the surviving copy as early as the eighth century CE and no later than the twelfth. Hoernle's dating, generated on the basis of a partial translation,

[2] *Whose name, incidentally, is incorrectly spelled in the reference list.*

relied on assumptions about the content, the symbolism, and the meter. Hayashi, who produced an authoritative transcription, translation, and commentary of the entire work, gives compelling reasons that rest on the media of the manuscript, paleographical evidence, and the language (a modified form of classical Sanskrit consistent with medieval northwest Indian vernaculars at that time). Given the uniqueness of this text, appeals to content for dating seem questionable, and one must be mindful not to conflate the stylistic features of a copy of an original with what the original may have looked like to establish dating. Choosing between scholarly evaluations of this sort requires a special sort of expertise. One needs familiarity with paleographical studies, epigraphy, codicology, as well as detailed knowledge of the text, to be able to appreciate the reliability of one set of evidence over another.

So which one does Joseph subscribe to? The issue of dating the Bakhshālī manuscript appears in at least three separate places in his survey (pp. 312, 317, and 358). The latter two references seem to be contradictory: on page 317 Joseph states, "On the basis of recent evidence, notably that of Hayashi, the manuscript cannot be dated earlier than the eighth century," and on page 358, "The general consensus supports Hoernle's dating [to the third century]."[3] Despite his apparent agreement with Hayashi at one point in his work, Joseph's contextualization and analysis proceed on the basis of Hoernle's dating.[4] The situation as Joseph portrays it is thus far from clear for the reader.

The popularity of *The Crest of the Peacock* rests upon Joseph's ability to synthesize scholarship that would otherwise be unpalatable for a general audience. His ample bibliography testifies to the broad range of sources on which his scholarship rests. However, all scholarly material included in the book, even that assumed to be common knowledge, still ought to be connected back to the original sources. For instance, Joseph's summary of Egyptian and Mesopotamian mathematics (pp. 181–183) relies point for point on the insights in Boyer's book, which goes unacknowledged by Joseph.[5] In another instance, Joseph refers to connections between Pānīni's grammar and the *Elements* of Euclid (p. 316) without acknowledging the promulgator of this theory, Frits Staal. Frequently invoked throughout the book (p. 103 and elsewhere) are Nesselman's criteria for the development of algebra (rhetorical, syncopated, symbolic), but Joseph does not indicate their connection to Nesselman. Out of these three instances, Boyer does appear in the general bibliography. The latter scholars do not. The reviewer provides the references here for those interested readers.[6]

Joseph relays some of the questions that should be addressed when dealing with early mathematical texts. He lists these (p. xx):

1. *What* was the content of the mathematics known to that culture?
2. *How* was that mathematics thought about and discussed?
3. *Who* was doing the mathematics?

These are good guidelines for approaching a historical document. To endorse this sentiment and complement the work done by Joseph, we offer some additional remarks to one of his examples, the history surrounding the emergence of triangular tables of binomial coefficients and associated mathematical relationships. This is to highlight the complexity of the task of addressing these questions as well as to emphasize that fully appreciating a development of this kind requires a synthesis of "western" (Greek) and "non-western" (Islamic, Chinese, Indian) approaches and sources rather than focusing exclusively on one group or the other.

Joseph discusses an instance of triangular table of binomial coefficients (sometimes known as Pascal's triangle) in his chapter on China, where he links the first explicit discussion of it to an early eleventh-century Chinese mathematician Jia Xian (whose work is lost but is discussed in a work of

[3] *The situation is not made much clearer by his summary (p. 367), which says, "The state of Indian mathematics at the middle of the first millennium AD, as represented by the Bakhshālī Manuscript…"*

[4] *Joseph also comments (p. 358) that there is no clue as to who was the author of the work. This is not entirely true. In fact, some prosopographical details do exist. The Bakhshālī manuscript contains a colophon that reveals that it was composed by a Brāhmaṇa who was the son of Chajaka, who wrote it for Hasika the son of Vasiṣṭha and his descendants. We cannot be sure, however, whether he was the author or the scribe, and nothing more is known about these individuals.*

[5] *See Carl B. Boyer,* A History of Mathematics, *2nd edition, John Wiley & Sons, New York, 1989, pp. 41–42.*

[6] *Frits Staal, "Euclid and Pānīni",* Philosophy East and West, *5.2 (1965), 99–116; J. Bronkhorst, "Pānīni and Euclid: reflections on Indian geometry",* Journal of Indian Philosophy ***29*** *(2001), 43–80; G. H. F. Nesselman, Versuch einer kritischen Geschichte der Algebra, vol 1.: Die Algebra der Griechen, (Reimer Berlin, 1842. Reprint by Minerva, Frankfurt, 1969). Furthermore, Joseph's preface discusses translation issues and invokes distinctions between "alienating" or "literal" translations with "user friendly" ones. This distinction refers to the work of Høyrup as contrasted with van der Waerden and Neugebauer and was discussed in detail by Eleanor Robson,* The Mathematics of Ancient Iraq: A Social History, *Princeton University Press, 2008, pp. 274–284.*

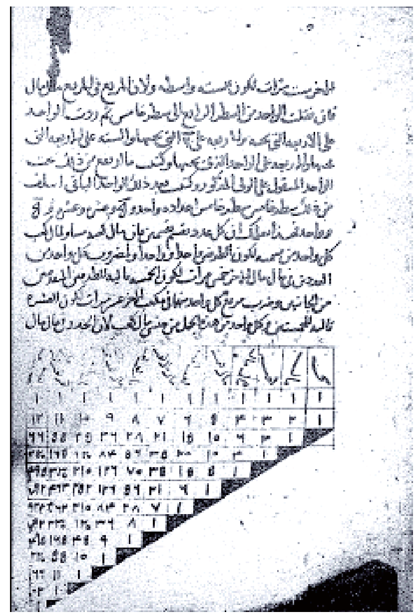
7	6	5	4	3	2	1	0	1	2	3	4	5	6	7
square square cube	cube cube	square cube	square square	cube	square	*thing*	unit	part *thing*	part square	part cube	part square square	part square cube	part cube cube	part square square cube
128	64	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}\frac{1}{4}$	$\frac{1}{4}\frac{1}{8}$	$\frac{1}{8}\frac{1}{8}$	$\frac{1}{4}\frac{1}{4}\frac{1}{8}$
2187	729	243	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{3}\frac{1}{9}$	$\frac{1}{9}\frac{1}{9}$	$\frac{1}{3}\frac{1}{9}\frac{1}{9}$	$\frac{1}{9}\frac{1}{9}\frac{1}{9}$	$\frac{1}{3}\frac{1}{9}\frac{1}{9}\frac{1}{9}$

Table 1. Al-Samaw'al's table of powers. The factorizations in the last rows are explicitly given by al-Samaw'al also.

Yang Hui around 1261).[7] In this context, he argues, the table of binomial coefficients appears to be a byproduct of exploring methods to extract square and cube roots. Yang Hui tells us that Jia Xian wrote a table of binomial coefficients in a triangle resembling the Pascal triangle up to the sixth row.

However, the exploration of binomial coefficients by early thinkers, their arrangement and manipulation in triangular arrays, and the ways in which they were thought about and used comprise a much more complex and nuanced theme in the history of mathematics. For instance, about the same time, or even a little earlier, the Islamic scholar al-Samaw'al completed a work called *Al Bāhir fī al-Jabr*, in which he set out rules in a rhetorical form for expanding expressions equivalent to $(ab)^n$ and $(a+b)^n$ for the cases $n = 3, 4$, extending his account to expansions for higher powers.[8] Al-Samaw'al credits this part of his work to his predecessor al-Karajī (953–c. 1029). He also includes in his manuscript a triangular table of binomial coefficients as well (see Figure 1).

Joseph's three questions concerning content, scope, and practice in this case can be properly appreciated only by considering the wider context of these achievements and the ways in which other cultures of inquiry had impacted the tradition al-Samaw'al and his predecessors were working in. For one, al-Samaw'al is indebted to his Greek predecessors in ways that he explicitly acknowledges. However, his work epitomizes the decisive breaks from Greek practice that were being advanced amongst Islamic scholars at that time.[9] Also of relevance are the new and significant use of diagrams in the text and an increasingly abstract articulation of number. All of these features are critical to understanding the role and function of this table in this context. Furthermore, this passage has already attracted much scholarly interest because of its relevance to the history of mathematical induction, a technique that finds antecedents in many different cultures of inquiry.



Courtesy of Turkey Manuscripts Institution, Süleymaniye Library.

Figure 1. The table of binomial coefficients of al-Samaw'al.

Al-Samaw'al's discussion furthermore reveals how rules for laws of indices were to be manipulated and used in this context. This is contrary to Joseph's belief that in general "without convenient notation for indices the laws of indices cannot be formulated precisely" (p. 351). When one consults al-Samaw'al's text, one can immediately appreciate that al-Samaw'al has various rhetorical equivalents for his algebraic powers, and he has a clear conception of the relation between successive

[7] *It may be relevant to note the activity in India by various earlier authors. See, for instance, the contributions in Robin Wilson and John J. Watkins (eds.),* Combinatorics: Ancient and Modern, *Oxford University Press.*

[8] *This passage has been thoroughly analyzed in an upcoming publication by S. Bajri, J. Hannah, and C. Montelle.*

[9] *Joseph does refer to al-Ṭūṣī's table, which appeared in 1265, and further in the book in an endnote (p. 302, no. 8), he notes the appearance "later" in Samarquand (does he mean al-Ṭūṣī?), and on p. 507 and p. 517, no. 35, links Pascal's triangle to al-Karajī and al-Samaw'al.*

powers (see Figure 1),[10] as he presents a table of them:

While their recursive relationship is not visible numerically (by means of numerical indices or otherwise) the organization of the table reveals a sympathy for their mutual relations into successively increasing powers. To the modern eye, this relationship is effectively hidden by having the powers expressed as the appropriate combinations of the words "square" and "cube". However, it was perfectly understood by the actors of the time. The alignment and arrangement of the table reinforced the relations of successive powers. This use of "square" and "cube" is clearly a remnant of the Euclidean geometric tradition and in this context reveals how transitional al-Samaw'al's mathematical practice is. Thus by considering how the text was read and used by those within the tradition and, more generally reflecting on the ways in which it resembles as well as contrasts with other articulations such as Indian, Chinese, Italian or indeed Blaise Pascal's account itself, we get a much richer sense of what it was like for early thinkers to investigate and articulate mathematical results such as these.

No doubt Joseph's work will continue to enjoy its original popularity; it represents the importance and impact the history of mathematics can have when brought to mainstream audiences. While it is not always a reliable substitute for specialist research, the book certainly gives avid readers a taste of the scope and breadth of aspects of early mathematical cultures of inquiry.

[10] *From S. Ahmad and R. Rashed,* Al-Bāhir en Algèbre d'As-Samaw'al, *Imp. de l'Université de Damas, 1972, p. 21.*